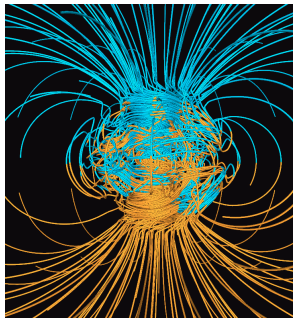


# Parallel in time integration of kinematic dynamos

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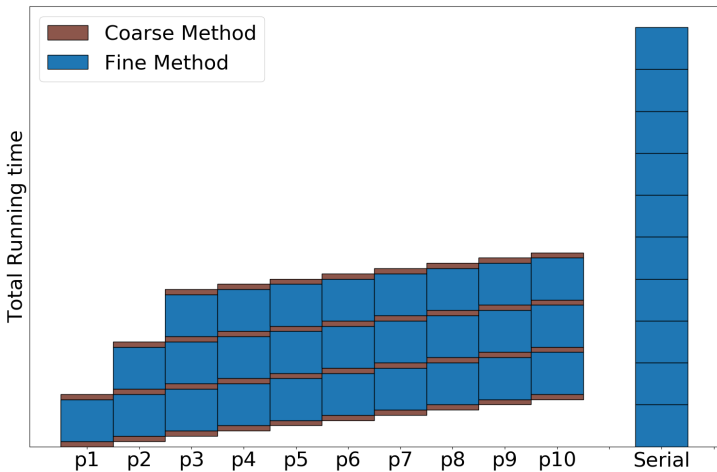
University of Leeds



This work was undertaken on ARC3, part of the High Performance Computing facilities at the University of Leeds, UK.

# Parallel in time: Parareal Algorithm

$$U_{n+1}^{k+1} = \underbrace{\mathcal{G}_{\Delta t}(U_n^{k+1})}_{\text{Coarse integrator}} + \underbrace{\mathcal{F}_{\delta t}(U_n^k)}_{\text{Fine integrator}} - \underbrace{\mathcal{G}_{\Delta t}(U_n^k)}_{\text{Coarse integrator}} \quad (1)$$



# Convergence

- Important relative quantity
- Examples:  $L^2$  Norm, integral quantities etc...
- Continue iterations until differences in quantities fall below tolerance.

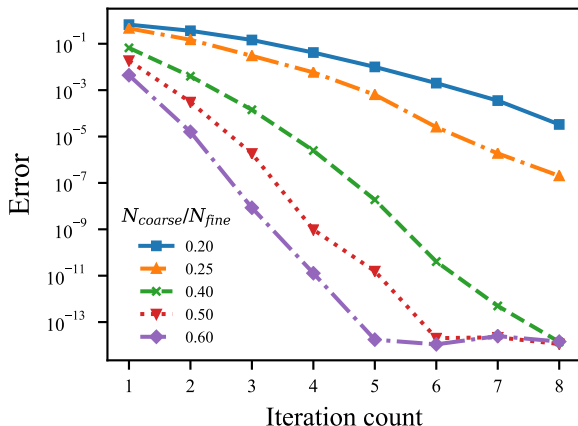
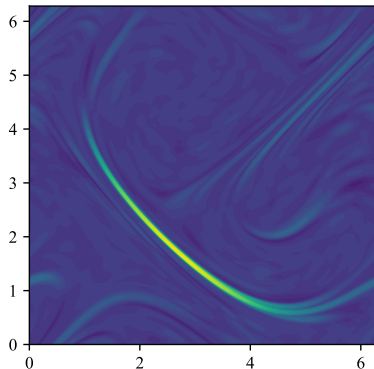
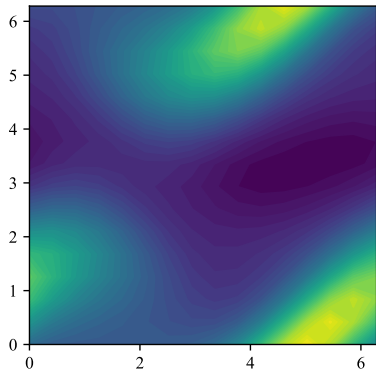


Figure 2: Impact on convergence of Coarse/Fine grid ratio

# Kinematic Dynamo

- Model for how magnetic fields generated in Sun/Stars/Earth
- Induction Equation:  $\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + R_m^{-1} \nabla^2 \mathbf{B}$
- Time dependent exponential growth in field strength
- Solved using Dedalus spectral solver, Python + MPI + FFTW



# Results

- Scaling extended past spatial saturation
- Speed ups  $\sim 300$  with 1600 processors
- Efficiency close to Parareal theoretical bound  $1/k$

